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Book Review: *Lattice-Gas Cellular Automata*

Lattice-Gas Cellular Automata. D. H. Rothman and S. Zaleski, Cambridge University Press, Cambridge, 1997.

This volume is subtitled "Simple Models of Complex Hydrodynamics." Lattice-based approaches to modeling fluid dynamics started to become popular in the mid-eighties; while some of the initial claims were, as some readers will surely remember, just a little over-optimistic, the methodology has proved highly successful in a number of areas. This book, by two well-known contributors to the field, is both an introduction to the methodology and a survey of several kinds of application of these methods to what are truly complex hydrodynamic problems.

The first part of the book begins with a historical introduction to lattice gases and cellular automata, then continues with a review of the assumptions underlying the conventional continuum treatment of fluid dynamics, an introduction to lattice-gas dynamics based on Boolean variables, and an examination of the relationship between microscopic dynamical quantities and microscopically averaged fields.

A detailed treatment of the hydrodynamics of invicid 2D lattice gases (LG) follows. Collision rules (these define the collision operator) are described in detail, together with a discussion of the need for a lattice which assures isotropy of the model. Viscous 2D hydrodynamics is studied, and the computation of the viscosity coefficient from the collision operator demonstrated.

The text then progresses to simple 3D models. Here it becomes clear why the LG approach is not as promising as was initially thought: the absence of adequate symmetry in familiar 3D lattices necessitates the use of a projected 4D lattice (with a coordination number of 24). The simplicity of the 2D case is misleading, since the 3D version is considerably more complex from a computational point of view.

The lattice-Boltzmann (LB) method is then introduced. This is also a lattice-based approach in which the Boltzmann equation itself is solved:

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the Boolean variables of the LG have been replaced by floating-point quantities representing the local population densities of particles with particular velocity directions. The approach is presented as a tool for computational (continuum) fluid dynamics; practical aspects are addressed, including the question of length scales, the implementation of boundary conditions, and issues of stability and convergence.

The second section of the book deals with more complex systems, in particular multicomponent fluids. The discussion begins with miscible fluids in which the type (or "color") of the particles is represented by means of a passive scalar quantity, and the problem is analyzed using both LG and LB techniques. The immiscible 2D LG follows. Here the collision rules depend not only on the incoming particle velocities but also on the particle types, so clearly the combinatorics of the collision process becomes more complex. The model is now sufficiently rich to allow the study of phase transitions and surface tension. The LB approach can also be used here, because appropriate modification of the collision operator produces an effective surface tension. Specialized methods are needed for a 3D immiscible LG to ensure a tractable computation; these techniques are discussed, and the drag force on a bubble evaluated.

A model for studying liquid-gas behavior is then introduced. Only a single particle species is involved, but it is nevertheless possible to achieve coexisting high and low density phases. In real fluids, this phase separation is a direct consequence of attractive interatomic forces; in order to obtain an analogous effect within the LG framework it is necessary to introduce additional collisions between certain pairs of nonadjacent particles ("collisions at a distance").

The final section of the book includes a number of applications in which the LG approach has proved particularly useful, with emphasis on problems to which the authors themselves have contributed. The subjects covered include single- and two-phase flow through porous media, the dynamics of phase separation, interfaces and their fluctuations, and a brief survey of more complex fluids including microemulsions. Theoretical topics are also covered here, namely the statistical-mechanical foundations of the LG approach, and a derivation of the Navier-Stokes equation for the model.

A series of mathematical appendices complete the work. Some of the chapters include exercises, and solutions to a few of the problems also appear here. A particularly nice feature of this book is the annotated bibliography; this should be of great help for readers interested in exploring the subject in greater depth.

The book is aimed at beginning graduate students, and readers at this level should be able to learn a lot from the material presented. It must of course be remembered that the LG and LB models were designed to serve

as computational tools for solving problems in fluid dynamics. The computational side of the subject is mentioned only in passing, and the book lacks descriptions of algorithmic details and software. The authors have, however, made some of their software available on a web site. This software covers several of the 2D systems described in the book, but will require a certain amount of effort on the part of the reader to convert the output into a usable (for example, graphical) format.

It is worth recalling, as the authors point out, that the birth of lattice-based hydrodynamics was announced on the front page of a prominent American newspaper. If the early claims of the LG community had proved correct, it would today be the only(!) technique used. But nature is more subtle, and while LG methods have found their niche, the more conventional finite-difference and finite-element methods–even though they are blissfully unaware of the atomistic nature of matter–still dominate. Time has shown that sacrificing physical reality on the altar of computational simplicity sometimes loses too much of the physics (and might also not provide the computational benefits originally envisaged).

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